## Phenomenology at colliders (3)

P. Marage Université Libre de Bruxelles Egyptian School on High Energy Physics BUE – Cairo –May 27 to June 4, 2009

## Plan

#### I. INTRODUCTION AND MOTIVATION

#### **II. STRUCTURE FUNCTIONS AND PARTON DISTRIBUTION FUNCTIONS**

- 1. Deep inelastic scattering and structure functions
- 2. Quark parton model
- 3. Scaling violation
- 4. QCD evolution and DGLAP equations

#### **III. FACTORISATION THEOREMS; PDF PARAMETERISATIONS**

- 1. Factorisation theorems
- 2. Drell-Yan production with CMS
- 3. Parton distribution function parameterisations
- 4. Parton distribution uncertainties
- 5. Some (of many) uncovered topics

#### **DIS cross section**

$$F_{1}(x,Q^{2}) = MW_{1} \qquad F_{2}(x,Q^{2}) = vW_{2}$$

$$\frac{d^{2}\sigma}{dxdy} = \frac{d^{2}\sigma}{dxdQ^{2}}xs = \frac{4\pi\alpha^{2}}{Q^{4}}s\left[(1-y)F_{2}(x,Q^{2}) + \frac{y^{2}}{2}2xF_{1}(x,Q^{2})\right] \qquad \text{em interaction : NC } \gamma \text{ exchange}$$

$$= \dots \left[\dots \pm \frac{G_{F}^{2}}{8\pi^{2}}\frac{Q^{4}}{(1+Q^{2}/M^{2})}y(1-y/2)xF_{3}(x,Q^{2})\right] \qquad \text{weak interaction : CC } W \text{ exchange}$$

#### $F_1$ , $F_2$ , $F_3(x,Q^2)$ = structure functions – physical observables (measured quantities)

#### Scaling

Incoherent scattering on free partons

$$\rightarrow F_2(x) = \sum_i e_i^2 x f_i(x)$$

$$\rightarrow F_1(x) = \frac{1}{2x} F_2(x)$$

Structure functions depend only on x; cross section given by quark distributions f(x)

$$\frac{d^2\sigma}{dxdy} = \frac{2\pi\alpha^2}{Q^4} s \left[ 1 + (1-y)^2 \right] \sum_i e_i^2 x f_i(x) \qquad \text{QPM}$$

#### **Scaling violations**

Q<sup>2</sup> evolution of structure functions

photon resolution improves with  $Q^2$  $\rightarrow$  disentangles virtual gluon emission



As Q<sup>2</sup> increases,

quark content decreases at large x (valence) and increases at low x

also : at low *x*, the gluon content and the sea increase

```
(low x since due to bremsstrahlung \rightarrow soft)
```

#### parton distribution function evolutions



#### « structure of the quark »



 $\hat{\sigma}_{\tau}(\mathbf{z}, \mathbf{Q}^2)$  is the photon-quark transverse cross section,

for a (« secondary ») quark of momentum fraction z;

 $\xi$  and z can vary from 0 to 1, but  $x = \xi z$  is fixed (hence the  $\delta$  function)

After integration on *z* :

$$2F_1(x,Q^2) = \sum_i \int_0^1 \frac{d\xi}{\xi} f_i(\xi) \frac{\hat{\sigma}_{\tau}(x \mid \xi, Q^2)}{\hat{\sigma}_0}$$

#### quark evolution equation

At first order :  $\gamma^* q \rightarrow q$  where  $z = x / \xi = 1$ 

At next order, the photon quark cross section contains a  $\gamma^* q \rightarrow q g$  contribution

with for 
$$\frac{d\hat{\sigma}}{dp_T^2} \simeq e_q^2 \hat{\sigma}_0 \frac{1}{p_T^2} \frac{\alpha_s(Q^2)}{2\pi} P_{qq}(z) \qquad \text{where } P_{qq}(z) = \frac{4}{3} \left(\frac{1+z^2}{1-z}\right)$$

 $P_{qq}(z)$  is the probability of a quark emitting a gluon and reducing the quark momentum by the factor *z* : « <u>splitting function</u> »

$$\hat{\sigma}(\gamma^* q \to qg) = \int_{\mu_F^2}^{s^2/4} dp_T^2 \frac{d\hat{\sigma}}{dp_T^2} \simeq e_q^2 \ \hat{\sigma}_0 \ \frac{\alpha_s(Q^2)}{2\pi} P_{qq}(z) \ \log \frac{Q^2}{\mu_F^2} \qquad \text{log. scaling violation}$$

Keeping the relation between  $F_2$  and quarks

$$\frac{1}{x}F_2(x,Q^2) = \sum_q e_q^2 \left[q(x) + \Delta q(x,Q^2)\right]$$

=> quark density evolution

$$\frac{dq(x,Q^2)}{d\log Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{d\xi}{\xi} q(\xi,Q^2) P_{qq}(\frac{x}{\xi})$$

#### **DGLAP** equations



Notation  $P_{ij} \otimes f_i(x,Q^2) = \int_x^1 \frac{d\xi}{\xi} P_{ij}(\frac{x}{\xi}) f_i(\xi,Q^2)$ 

$$\frac{dq(x,Q^2)}{d\log Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \Big[ P_{qq} \otimes q(x,Q^2) + P_{qg} \otimes g(x,Q^2) \Big]$$
$$\frac{dg(x,Q^2)}{d\log Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \Big[ P_{gq} \otimes q(x,Q^2) + P_{gg} \otimes g(x,Q^2) \Big]$$

#### Remarks

1. DGLAP equations = Renormalisation group equations (RGE)

$$q(x, Q^2; \mu_F^2) = q(x) + \frac{\alpha_{S}(Q^2)}{2\pi} \log \frac{Q^2}{\mu_F^2} \int_X^1 \frac{d\xi}{\xi} P_{qq}(\frac{x}{\xi}) q(\xi)$$

Choice of factorisation scale  $\mu_F$  is arbitrary  $\rightarrow q(x, Q^2)$  should not depend on  $\mu_F$ :

$$\frac{dq(x,Q^2;\mu_F^2)}{d\log\mu_F} = 0 \quad \rightarrow \text{ the DGLAP equations}$$

#### 2. Higher orders

NLO and NNLO splitting functions have been calculated. Very complicated !

III. Factorisation theorems; pdf parameterisations

## **III.1 Factorisation theorems**

#### Infrared singularities

Remember logarithmic singularity for quark structure, due to collinear gluon emission

$$\hat{\sigma}(\gamma^* q \to qg) = e_q^2 \ \hat{\sigma}_0 \ \frac{\alpha_s}{2\pi} P_{qq}(z) \ \log \frac{Q^2}{\mu_F^2} + \int_0^{\mu_F^2} dp_T^2 \ \frac{d\hat{\sigma}}{dp_T^2}$$

For gluon structure, log (Q / m) singularity due to  $\gamma g$  fusion diagrams



Generally speaking, *infrared* singularities due to *soft* and *collinear* configurations

(degenerate kinematic situations)

they correspond to on mass shell intermediate parton, with  $k^2 = m^2 \approx 0$ 



They correspond to *long distances* 

#### **QCD** factorisation theorems

(to be demonstrated : DIS, jet production, Drell-Yan, prompt photon emission, fragmentation in  $e^+e^-$ ) :

#### Infrared (long distance) singularities (due to nearly on mass shell partons) can be separated from hard (short distance) partonic process (with large off mass shellness)

i.e. infrared singularities can be « factorised out »

order by order in pQCD (or useless !)

into *universal* parton density functions

- which must be *measured* (cannot be calculated !)
- at some factorisation <u>scale</u>  $\mu_F$
- of which the <u>evolution</u> from  $\mu_F$  can be calculated using the  $P_{ij}$  coefficient kernels

(DGLAP equations)

Very much like charge and mass are redefined to dispose of familiar UV singularities due to loop corrections

« renormalisation » is factorisation of UV divergences

« factorisation » is renormalisation of soft / collinear divergences

#### Master formula

ones often takes  $\mu_F = \mu$  - can be  $Q^2$  or  $E_T$  (jet) etc.

NB complicated cases where 2 scales (e.g.  $Q^2$  and jet  $E_T$ ; also when large log 1/x)

 $\blacktriangleright$  the factorisation scale  $\mu_F$  can be seen as where hard and soft processes separate,

i.e. maximum off-shelness of partons grouped into pdf  $\phi_{i/h}$ 

> as  $\mu$  is present in both coeff. fct. and in pdf's,

a « factorisation scheme » (*MS-bar*, *DIS*) must define (for higher orders) the attribution of the short distance finite contributions (i.e. to coeff. fct. or to pdf's) (remember : pdf's are « theoretical » objects)

#### **Parton distribution functions**

$$\sigma^{h}(x,Q^{2}) = \sum_{i=q \ \bar{q} \ g} \int_{0}^{1} \frac{d\xi}{\xi} \quad C^{i}(\frac{x}{\xi},\frac{Q^{2}}{\mu^{2}},\frac{\mu_{F}^{2}}{\mu^{2}},\alpha_{S}(\mu^{2})) \quad \phi_{i/h}(\xi,\mu_{F},\mu^{2})$$

 $\Box$  coeff. functions are QCD calculable as power series in  $\alpha_s$ ,

infrared safe process dependent (NC DIS, CC DIS, jet, etc.) independent of initial hadron *h* 

□ pdf's are specific to *h* 

but process independent (including independent of  $Q^2$ )

□ pdf evolution kernels (e.g. DGLAP) are

QCD calculable as power series in  $\alpha_s$ 

infrared safe

- > compute the process (e<sup>+</sup> e<sup>-</sup>, DIS, …) cross section at parton level, at a given order of perturbation theory
- > compute the « parton structures »  $\phi_{i/q} \phi_{i/q}$  at the same order (in a given factorisation scheme)
- $\succ$  thus derive the coefficient functions  $C^i$  (at same order, in the same scheme)
- > combine the  $C^i$  with the experimental cross section  $\sigma^h$  to derive the non perturbative parton distributions in the hadron  $\phi_{i/h}$  (at same order, in the chosen scheme) (i.e. inverse master formula)
- $\blacktriangleright$  use the evolution kernels to extract the pdf's for a given  $\mu$  factorisation scale value

# III.2 Drell-Yan production with CMS

### **Drell-Yan production**





LHC 
$$q\bar{q} \rightarrow \gamma / Z \rightarrow e^+ e^-$$





## **Kinematics**

quark with proton energy fraction  $x_1$  antiquark with  $x_2$ 

Let us compute

 $M = \sqrt{(x_1 x_2)} \sqrt{s} \quad (\sqrt{s} = 2E_b)$ 

 $x_1 x_2$  not fixed and no reason that  $x_1 = x_2$ 

i.e. two interacting particles (quarks) have different energies  $\neq e^+e^-$ M = 100 GeV  $\rightarrow \langle x \rangle = ?$ 

but mass distribution depends on quark distribution in proton - pdf's





NB different acceptance for e+ and e-

#### Different acceptance for electrons (solid) and positrons (dashed)

In SM, e<sup>-</sup> is preferentially emitted in direction of quark x(quark) is generally larger than x (antiquark) => e<sup>-</sup> is statistically more boosted than e<sup>+</sup>



#### Different acceptance for low (200 GeV - solid) and large mass (2000 GeV - dashed)

2000 GeV =>  $\sqrt{(x_1x_2)}$  = 0.2 => both quark at relatively large x => Z not much boosted 200 GeV =>  $\sqrt{(x_1x_2)}$  = 0.02 => x (quark) can be large (0.1), x (antiquark) small 0.004 => very different => Z boosted



(Master thesis V. Dero, ULB)

## jet production

Jets LO diagrams



different diagram contributions (gg, gq, qq) depend on pdf's

Tevatron qg dominate

LHC gg dominate





## top pair production



## underlying event (soft physics)

Electron identification against jet background : isolation criteria



« Hard »  $q\bar{q} \rightarrow \gamma / Z \rightarrow e^+ e^-$  interaction

+ proton remnant jets

+« soft » interactions between proton remnants = high density colour fields

-> additional tracks with limited  $p_{T}$ 



(Master thesis V. Dero, ULB)

Coupures	Nombre moyen de particules	
	A une masse de 200 $GeV$ A une masse de 2000 $GeV$	
Pas de coupure	359	351
$ \eta  < 2.4$	159	162
$p_t > 1 \ GeV$	58	63
$ \eta  < 2.4$ et $p_t > 1 \ GeV$	35	40



III.3 Parton distribution parameterisations

#### Parameterising pdf's

> Choose a starting parameterisation for the various parton species (quarks, antiquarks, gluons)

at a given  $\mu$  scale (usually  $\mu_F = \mu$ )

in a given factorisation scheme (usually *MS-bar*)

- with a number of parameters sufficiently *large* to describe the data
- but sufficiently <u>small</u> to be really constraint by physics and not artefacts
- > Decide upon simplification hypotheses to decrease number of degrees of freedom
  - isospin (u(x) in proton = d(x) in neutron; u sea in proton = d sea in neutron, but u sea in proton might be different form u sea in neutron)
  - x-distributions of quark and antiquark seas : have to be the same in total, but what about x dependences ?
  - s(x) sea versus u(x), d(x) seas
- Choose experimental data
  - theoretically relevant (be sure factorisation applies !)
  - theoretically under control e.g.

higher order effects (NLO / LO ; NNLO / NLO)

treatment of nuclear effects (in extracting neutron pdf's from eA and  $\mu$ A scattering)

experimentally reliable

(for errors - see below !)

(e.g. phase space extrapolations for HERA charmed meson production)

> ... and fit



DIS (H1, ZEUS)

around 20 free parameters (or even more) for some 2000 data points

( $A_u$  and  $A_d$  fixed by valence quark counting,  $A_a$  fixed by momentum sum rule)

Parameterisations differ in detailed form of parameterisation at starting scale, data sets included, factorisation / renormalisation scale  $Q_0^2$  and scheme, value of  $\alpha_s(Q_0^2)$ , assumptions on  $\kappa$ , sea asymmetry, possible negative gluon

#### Data sets

**DIS** (1) fixed target  $\mu p, \mu n$  BCDMS, NMC, SLAC, E665  $x > 10^{-2}$   $e^+ p, e^- p$  (NC and CC) H1, ZEUS  $x > 10^{-5}$  quarks, gluons (through evolution)  $e^+ p, e^- p$  CC  $\rightarrow u / d$  at large x (without nucl. tgt problems)  $F_{cc}^2 F_{bb}^2 \rightarrow$  direct access to gluons (photon gluon fusion)

2009 joint analysis by H1 and ZEUS of 1995-2000 data set

110 point-to-point correlated error sources

 $\chi^2$  / dof = 576 / 592







Data sets (2)

**DIS** (2)  $vp vn \overline{v}p \overline{v}n$ 

CCFR $x > 10^{-2}$ : total quarks, valenceNuTeV+ strange sea (dimuon events from CC charm prod.)





Jets

Tevatron collider

Jets in DIS at HERA ZEUS

Sample of LO diagrams:









Data sets (5)

Prompt photon production

Jun lee

Sensitive to primordial  $k_T$  of quarks inside nucleon (i.e. higher orders



#### Results...

$$\frac{dq(x,Q^2)}{d\log Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \Big[ P_{qq} \otimes q(x,Q^2) + P_{qg} \otimes g(x,Q^2) \Big]$$
$$\frac{dg(x,Q^2)}{d\log Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \Big[ P_{gq} \otimes q(x,Q^2) + P_{gg} \otimes g(x,Q^2) \Big]$$



III.4 Parton distribution uncertainties

#### **Experimental uncertainties**

selection of data

choice of accepted  $Q^2$ , W domain

- effect of experimental errors ?
   correlated / uncorrelated systematics
- □ how to combine « poorely compatible » experiments ?
- Hessian estimate of errors (correlation matrix)

deviation in  $\chi^2$  of the global fit from the minimum  $\chi^2$  value is assumed to be quadratic in the deviation of the fitted parameters errors from their best value  $\rightarrow$  errors obtained from the covariance matrix, with  $\Delta \chi^2 = 1$ 

BUT - hypothesis on the quadratic behaviour of uncertainties : (very) questionable

- (there may exist) strong correlations between parameters (if larger number than necessary)

- inconsistencies between experiments

→ which tolerance to define errors on pdf's ?  $\Delta \chi^2$  = 100 (CTEQ), 50 (MRST), 1 (H1 – only DIS) ?

Lagrange multipliers : a series of global fits using Lagrange parameters attached to each given measurement, constraining the measured cross sections by the quoted errors → how does the global description deteriorates as one moves away from the unconstrianed best fit – while spanning a range of Lagrange multipliers

But very heavy procedure

#### **Theoretical uncertainties**

- □ higher QCD orders in DIS : NNLO
- $\Box$  log (1/x) and log (1-x) effects
- □ absorptive corrections parton recombinations
- other higher twist contributions
- □ form of the parameterisation at starting scale
- □ number of parameters ?
- □ ... and relevance of the chosen factorisation scheme for the chosen parameterisation form
- □ choice of starting scale of evolution
- $\Box$  choice of  $\alpha_{s}$
- simplification assumptions

isospin violation

 $S \neq \overline{S}$ 

- treatment of heavy flavours
- nuclear effects
- □ inclusion of e-w corrections (significant at NNLO)
- **D** ...

#### Remark : pdf's in Monte Carlos

Present Monte Carlos are generally LO + simulation of higher orders through parton shower (JETSET) JETSET follows DGLAP evolution – HERWIG is believed to be closer to BFKL evolution

#### **Higher orders**

All order summation is finite (factorisation theorem) but how fast is the convergence ?

trust convergence if corrections decrease when computing next order



sensitivity to scale = indication of size of next order contribution

$$\mu \frac{d}{d\mu} C^{(n)}(x, Q^2, \mu) \sim O(\alpha_s^{n+1})$$

small scale sensitivity at NL for DIS and D-Y large for heavy quarks and prompt photon

#### Heavy quarks

No HQ in the nucleon at small scale

dynamically generated (photon gluon fusion)



Works at not too large Q<sup>2</sup> but logarithmic divergence at large Q<sup>2</sup>  $\approx \log \frac{Q}{m_q}$ 

- $\succ$  at large Q<sup>2</sup>, treated as massless quarks
- $\rightarrow$  Fixed / variable flavour number scheme

#### **Jets**

full NNLO calculations not available yet

- $\rightarrow$  estimated through scale dependence :
  - $\mu$  often varied from 0.5  $E_T$  to 2  $E_T$

#### **Resummations**

- Fixed order calculations ←→ resummation of all order contributions : *leading logarithms* Necessary when 2 scales, e.g. Q<sup>2</sup> and jet E<sub>T</sub>
   ! double counting !
- DGLAP evolution : hard scale given by Q<sup>2</sup> resums α<sup>n</sup><sub>S</sub> log<sup>n</sup> Q<sup>2</sup> terms (+ NLO etc.), corresponds to strong ordering in k<sub>T</sub> of (virtual) partons
- ► BFKL evolution : in DIS domain (sufficiently large  $Q^2$ ), very high energy resums  $\alpha_s^n \log^n \frac{1}{x}$  terms corresponds to strong parton ordering in *x* (long. momentum) but not necessarily in  $k_T$

Predicts fast increase

CCFM evolution : connexion between DGLAP and BFKL angular ordering :  $\theta = \frac{k_T}{xp}$ 





III.5 (Some of many) uncovered topics

#### **Other parton distributions**

#### $\Box$ unintegrated $k_T$ distributions

relevant at very high energy, and when no strong  $k_{\tau}$  ordering (BFKL domain) e.g. large  $k_{\tau}$  jet or particle at large x  $Q^{2}$  x  $x_{n}, k_{T,n}$ Refere  $x_{n-1}, k_{T,n-1}$ Refere  $x_{1}, k_{T,1}$ Refere  $Q^{2}$   $Q^{2$ 

generalised parton distributions

correlations between partons



vector meson and real photon production (DVCS) most relevant for large mass difference between initial and final state

#### Spin parton distributions

dedicated experiments (HERMES, COMPAS, etc.)

#### Other hadrons or hadronic objects

#### photon

 $\gamma \gamma$  scattering at LEP, hard photoproduction at HERA

i.e. measurement of the hadronic structure of the photon

(« resolved » photon  $\leftarrow \rightarrow$  « direct » photon = pointlike)

 $\gamma \rightarrow q\bar{q}$  + evolution, including gluon content of the photon

NB in DGLAP evolution, inhomogeneous component (cf. NS SF)

#### pion

Drell-Yan, leading neutron final states at HERA (interactions on the pion virtual cloud around the proton)

pomeron : hadronic structure of diffractive exchange HERA (total diffractive production, vector mesons, charm, jets, etc. Tevatron (diffractive jet and W production) LHC : diffractive Higgs production



Factorisation theorem proved but strong higher twist contributions

- + effects on evolution equations
- + underlying interaction  $\rightarrow$  breaks simple application of of pdf transportation from HERA to Tevatron (« survival probability »)

#### Some references

- Introduction on DIS, SF, etc.
   F. Halzen, A.D. Martin, *Quarks and Leptons*, Wiley
- Introduction to pdf's and QCD

CTEQ site <u>http://www.phys.psu.edu/~cteq/</u> in particular QCD Handbook <u>http://www.phys.psu.edu/~cteq/#Handbook</u> W.K. Tung, Perturbative QCD and the parton structure of the nucleon see also : J.C. Collins, What exactly is a parton density? arXiv:hep-ph/0304122

- Present status of pdf's draw your favourite pdf's MRST site <u>http://durpdg.dur.ac.uk/hepdata/</u>
- Pdf uncertainties : see e.g. (+ ref. therein)
   A.D. Martin, R.G. Roberts, W.J. Stirling and R.S. Thorne Uncertainties of predictions from parton distributions
   I. Experimental errors arXiv:hep-ph/0211080
   II. Theoretical errors arXiv:hep-ph/0308087
- CERN PDFLIB manual <u>http://consult.cern.ch/writeup/pdflib/</u>